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Title: Lagrangian Particles in Ocean Modeling

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Lagrangian Particles in Ocean Modeling

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MPAS-Ocean *Eulerian* Model Equations

Primitive Equations: incompressible, hydrostatic, Boussinesq

Conservation Equations

mass (volume)

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) + \frac{\partial}{\partial z}(hw) = 0,$$

momentum
Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + q(h\mathbf{u}^{\perp}) + w\frac{\partial \mathbf{u}}{\partial z} = -\frac{1}{\rho_0}\nabla p - \nabla K + \nu_h(\nabla\delta + \mathbf{k}\times\nabla\eta) + \frac{\partial}{\partial z}\left(\nu_v \frac{\partial \mathbf{u}}{\partial z}\right),$$

tracers (temperature, salinity)

$$\frac{\partial h\varphi}{\partial t} + \nabla \cdot (h\varphi\mathbf{u}) + \frac{\partial}{\partial z}(h\varphi w) = \nabla \cdot (h\kappa_h \nabla\varphi) + h\frac{\partial}{\partial z}\left(\kappa_v \frac{\partial \varphi}{\partial z}\right)$$

Constitutive relations

hydrostatic in the vertical

$$\frac{\partial p}{\partial z} = -\rho g$$

equation of state

$$\rho = \rho(T, S, p)$$

Ringler et al. 2013, Ocean Modelling

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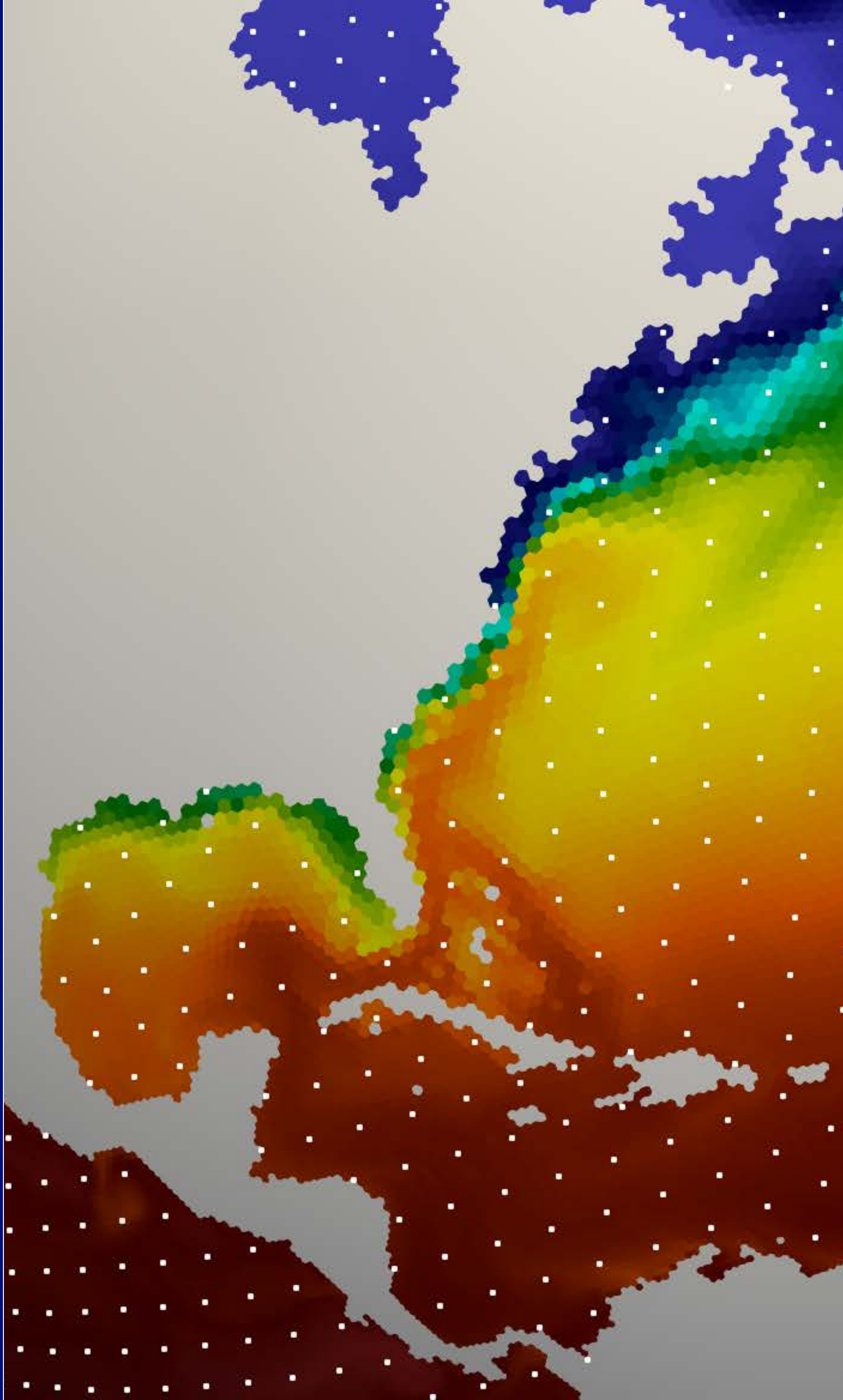
Lagrangian In-situ Global High-performance particle Tracking (LIGHT)

- Developed in 2015 by Phillip Wolfram
- Parallel particle tracking at exascale for climate simulation
- Vertical Motion
  - Fixed Depth
  - Adiabatic motion (isopycnal)
- Horizontal Motion
  - Interpolated from Eulerian grid

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{u}(\boldsymbol{x}, t)$$



EC30to60 mesh showing temperature and particle position.

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Why use Lagrangian Particles?

- Lagrangian particles are used to simulate data collection devices such as Argo, RAFOS, and Drifters.

- Useful for testing methods of data assimilation.

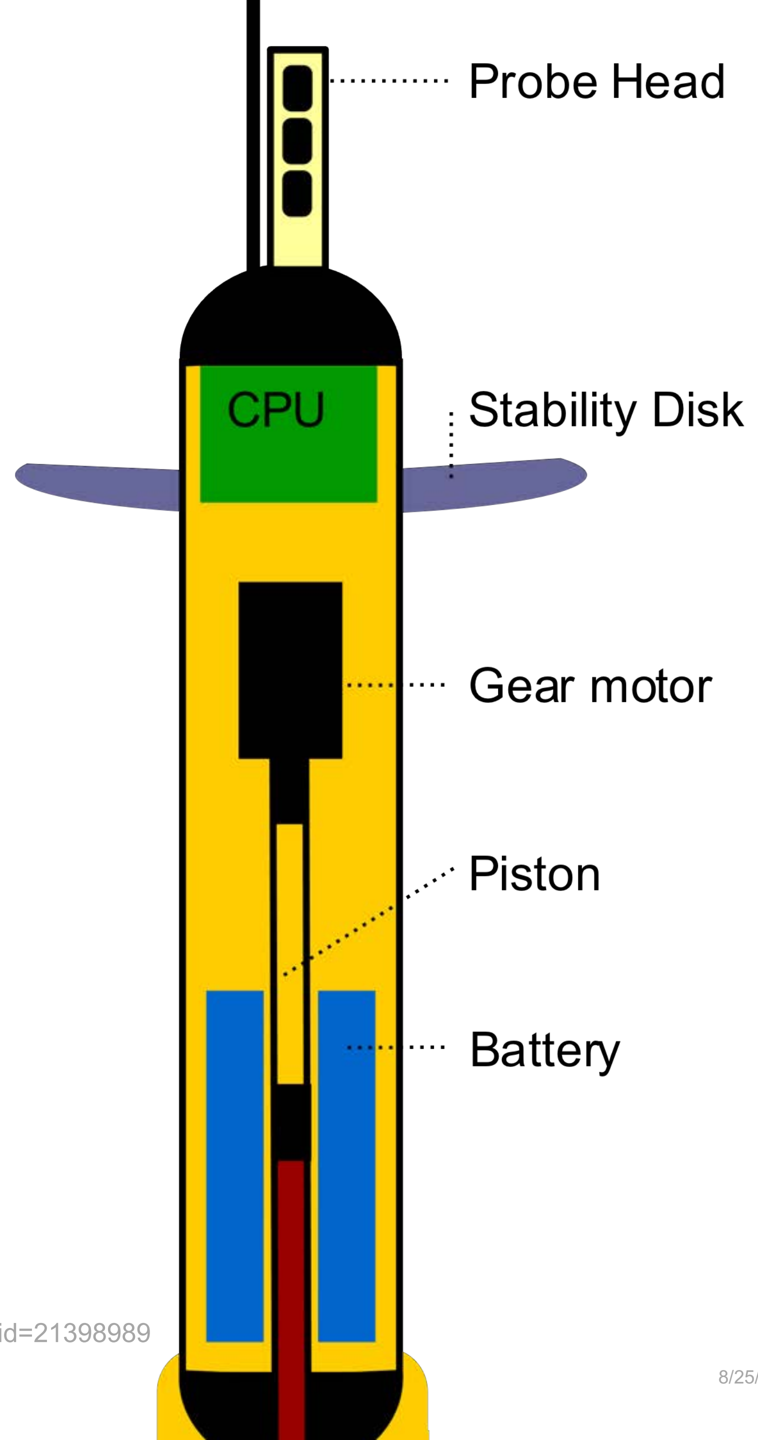


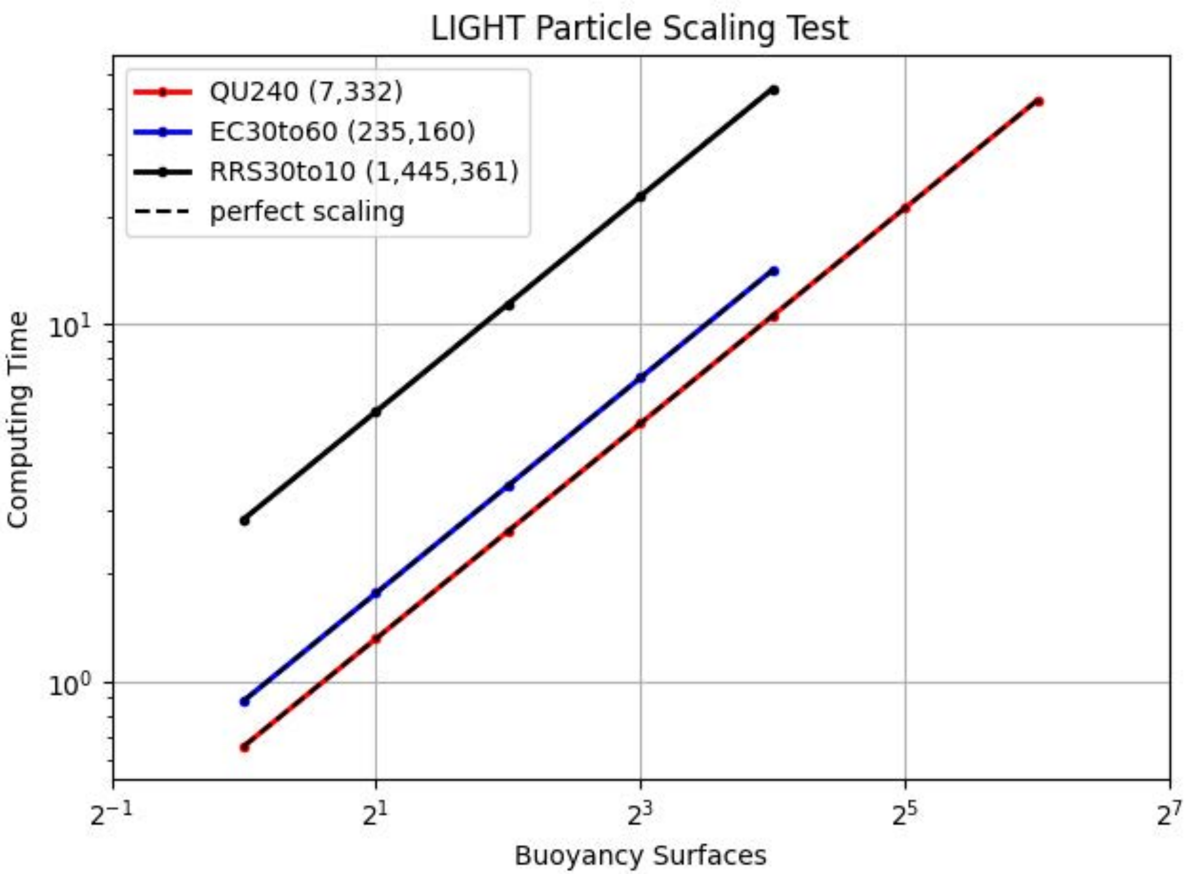
Image of Argo float.

Methodology

- Run baseline performance tests
  - Scaling as a function of number of particles.
  - Scaling as a function of number of cores.

- Convert per-particle variables to global flags
  - Original design was versatile but less performant.
  - Timestep, reset functionality

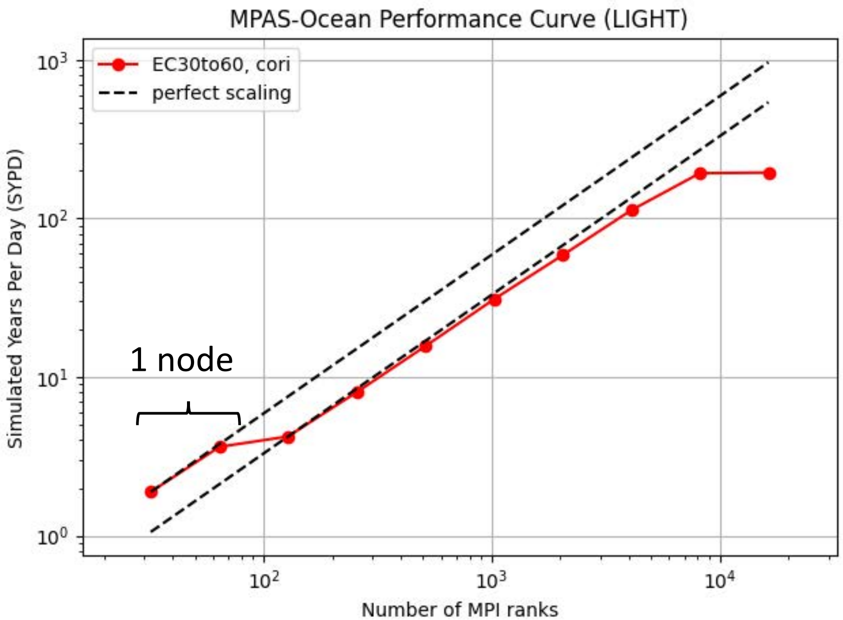
Results



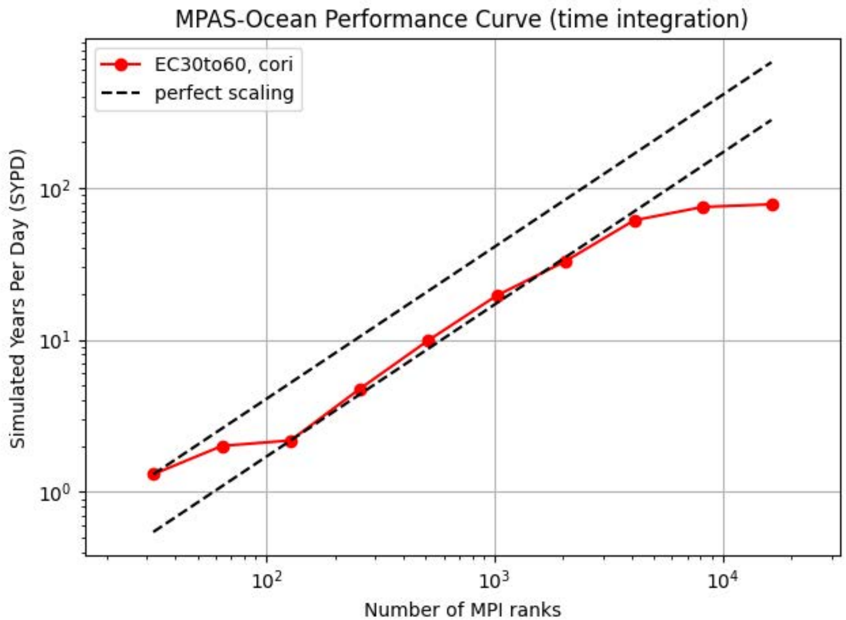
- Particles are suspended at different levels of buoyancy.
- Simulations used one particle per cell on each buoyancy surface.

Results

Strong Scaling Test, LIGHT Only



Strong Scaling Test, Full Ocean Model



- Test was run on cori-haswell at NERSC
- Run with 500k particles (2 per grid cell)

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Summer LIGHT Performance Results

- Creation of global flags produced a marginal speedup in running simulations with LIGHT, ~2%.
- Decreased the memory usage significantly for particles, ~15%.
- These are the results from my Parallel Computing Summer Research Institute Project.
- Will continue working through May 2022 on assimilation of particle data for DOE SCGSR program.

Data Assimilation

Consider a dynamical system of the form

$$\begin{cases} u_t = F(u), \\ u(0,x) = u_0(x) \end{cases}$$

Problem

Even if we can solve the equation, we don't know the initial data.

Continuous Data Assimilation

Simulation to obtain "observations".

$$\begin{cases} u_t = F(u), \\ u(0,x) = u_0(x) \end{cases}$$

Remember

- We don't know u or $u_0(x)$.
- We observe u at specific points.

Simulation to test data assimilation.

$$\begin{cases} v_t = F(v) + \mu \left(I_h(u) - I_h(v) \right), \\ v(0,x) = v_0(x) \end{cases}$$

Here, $\mu > 0$ is a constant relaxation parameter (units 1/time), and $I_h(f) = I_h(f, X)$ denotes the interpolation in space of $f = f(x,t)$.

Theorem (Azouani, Olson, Titi, 2014)

Let F be given by 2D Navier-Stokes equations.
For μ sufficiently large and h sufficiently small, $\|u(t) - v(t)\|_{H^1} \to 0$ exponentially fast as $t \to \infty$ for ***any*** *$v_0 \in L^2$, $\nabla \cdot v_0 = 0$.*

Interpolation Term

$$\begin{cases} v_t = F(v) + \mu I_{L\to E}\big(u - I_{E\to L}(v)\big), \\ v(0,x) = v_0(x) \end{cases}$$

Here, $\mu > 0$ is a constant relaxation parameter, with $I_{L\to E}$ and $I_{E\to L}$ are interpolation operators.

$I_{E\to L}(v)$ interpolates the value of v at the locations of the Lagrangian particles from the Eulerian grid.

$I_{L\to E}$ interpolates from the locations of the Lagrangian particles back to the Eulerian grid.

Brief Recent History of AOT Algorithm

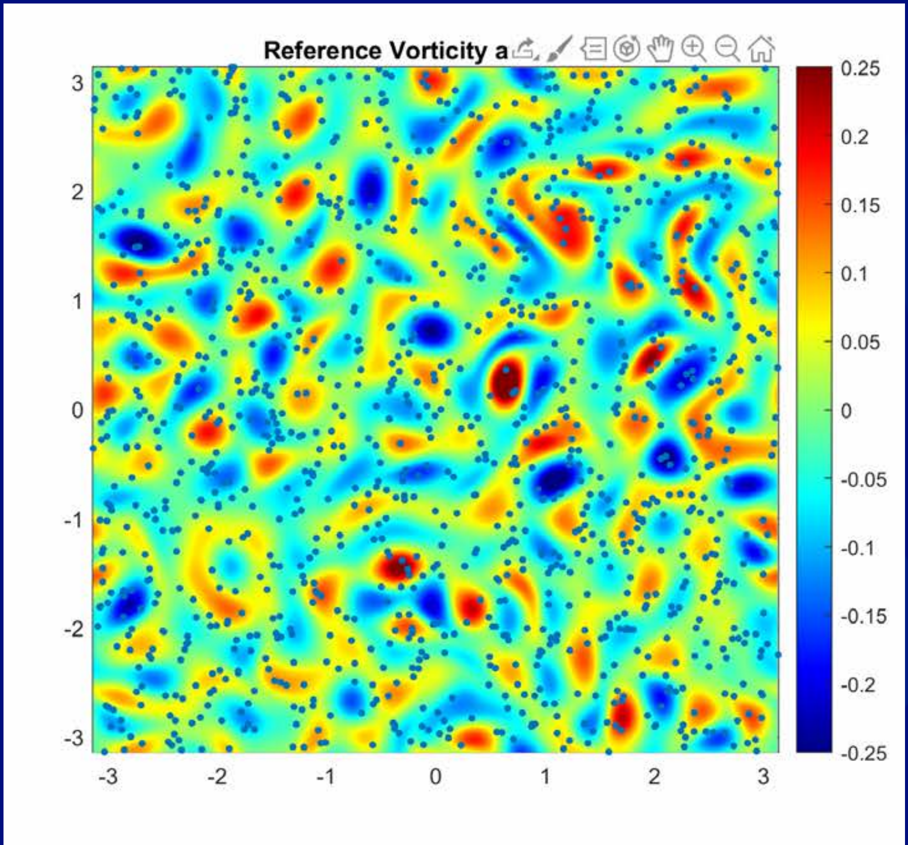
Development of Algorithm	Nudging (Anthes, 1974; Hoke, Anthes, 1976)
	Stabilization of NSE steady states (Cao, Kevrekidis, Titi, 2001)
	Determining Modes (Olson, Titi, 2003)
	Lorenz (Hayden, Olson, Titi, 2011)
	Reaction-Diffusion (Azouani, Titi, 2014)
	2D NSE (Azouani, Olson, Titi, 2014)
Recent Studies	2D simulations (Gesho, Olson, Titi, 2015)
	Stochastic noisy data (Bessiah, Olson, Titi, 2015)
	2D Abridged (Farhat, Lunasin, Titi, 2016)
	Discrete in time data (Foias, Mondaini, Titi, 2016)
	Statistical solutions (Biswas, Martinez, 2017)
	Fully Discrete Case (Mondaini, Titi, 2018; Larios, Rebholz, Zerfas, 2018)
	Parameter Recovery (Carlson, Hudson, Larios, 2018)
	Time averaged data (Jolly, Martinez, Olson, Titi, 2019)
	2D MHD using one component of velocity & magnetic fields (Biswas, Hudson, Larios, Pei, 2018)
	Dynamical downscaling of general circulation models (Desamsetti et al. 2019)
	2D NSE with local observers (Biswas, Bradshaw, Jolly, 2020)
	MPAS-Ocean mesoscale eddies (Carlson, Van Roekel, Godinez, Petersen, Larios, 2021)

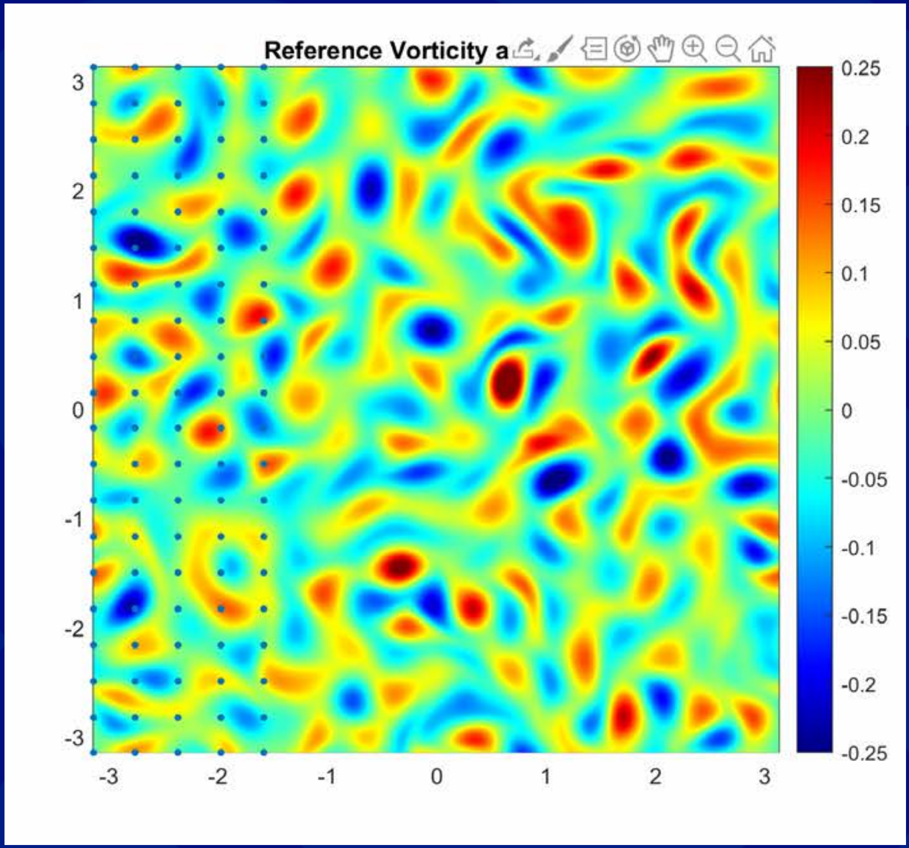
Mobile Observers in 2D Navier-Stokes

We studied the use of observers that move dynamically in time according to various movement patterns.

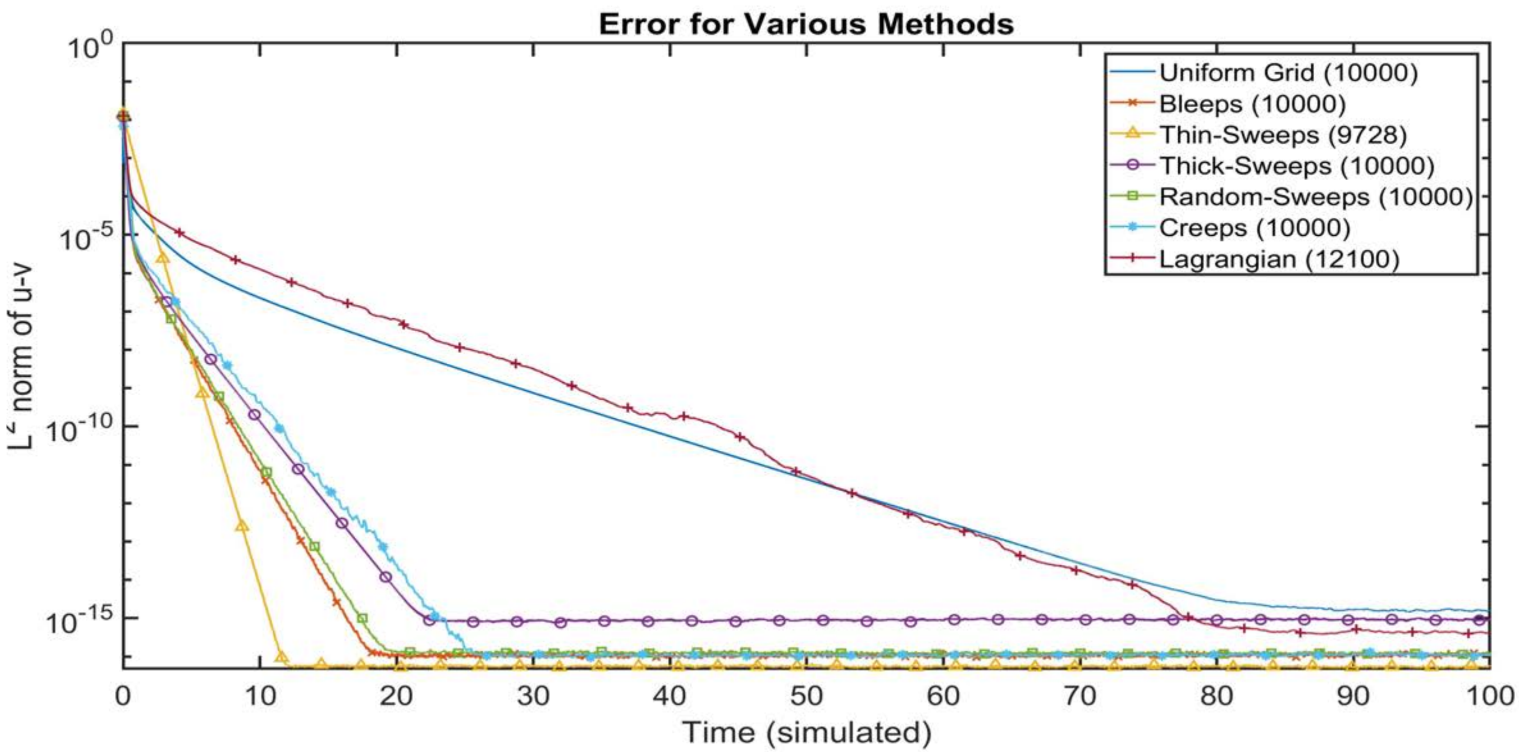
Bleeps	• Placed in random locations and moved frequently.
Sweeps	• Moving strips that measure full solution (Thin). • Moving uniform grid covering ¼ of domain (Thick). • Random locations with random velocities (Random).
Creeps	• Randomly walk around domain.
Lagrangian	• Follow Lagrangian particle trajectories.

Demo: Lagrangian and Thick-Sweeps





Exponential Convergence to Observational Data



Log-linear plot comparing convergence rates for methods initialized approximately the same number of observers.

- 2D NSE solved using pseudo-spectral methods with spatial resolution 1024^2.
- $\nu = 0.0001$ with forcing as in Gesho, Olson, Titi (2015).
- μ values were varied between methods.

Conclusions for 2D Navier-Stokes

- Exponential convergence of simulations to observations for all observer types.

- Mobile observer methods outperform static observers except in the case of Lagrangian particles.

- Manuscript nearly submitted on this work.

- Next, we will implement Lagrangian data assimilation using LIGHT in MPAS-Ocean to test these methods on a realistic ocean model.

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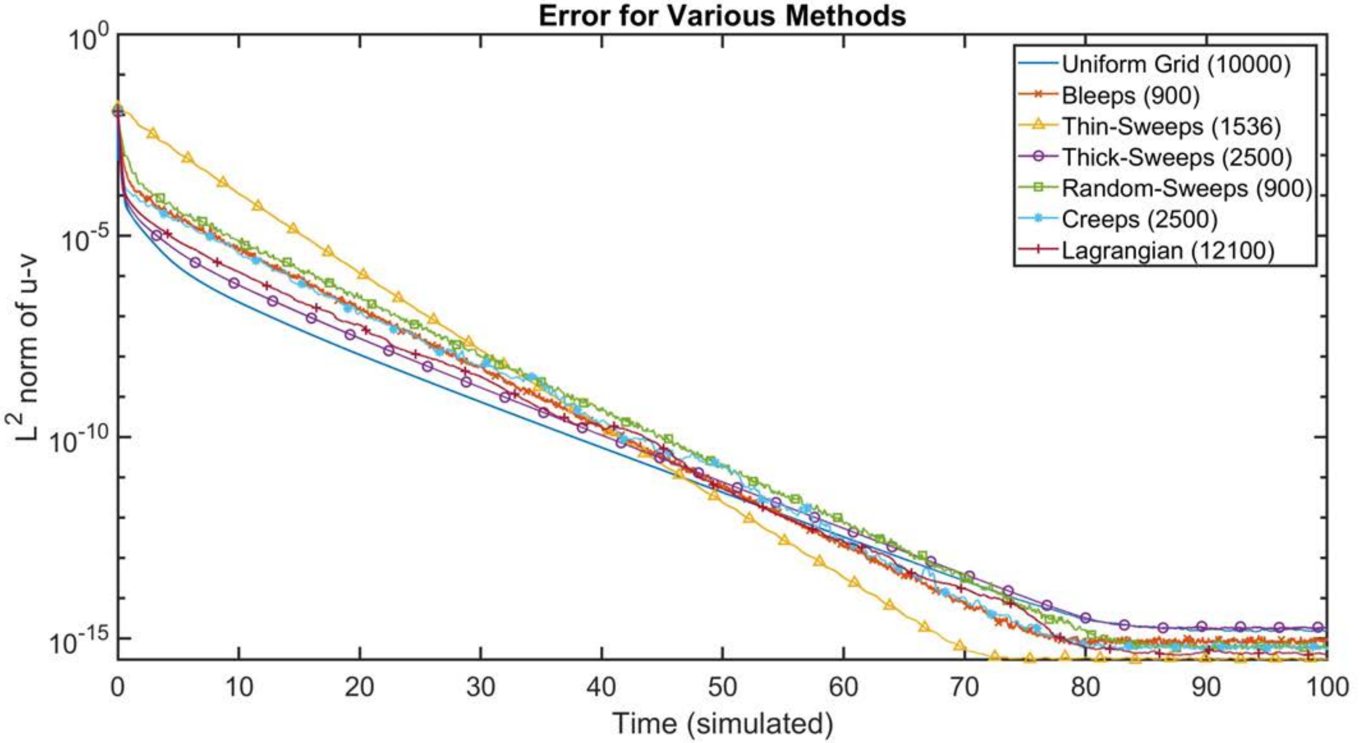
Exponential Convergence to Observational Data



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